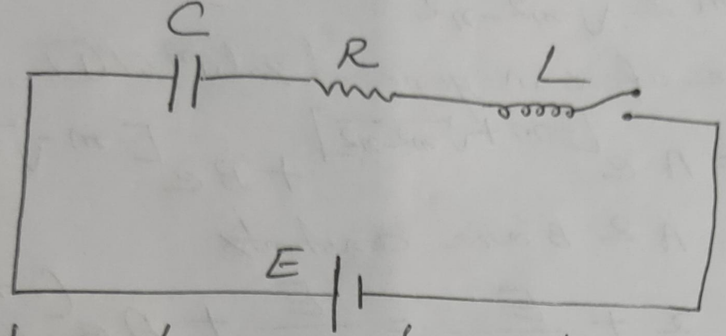
Q. Drive expression for growth of current in LCR Circuit.

Ans: →



By ohm's law, we have: →

$$E - L\frac{dI}{dt} - \frac{Q}{C} = RI = R\frac{dQ}{dt}$$

$$\Rightarrow E - L\frac{d}{dt}\left(\frac{dQ}{dt}\right) - \frac{Q}{C} = R\frac{dQ}{dt}$$

$$\Rightarrow \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{E}{L}$$

or, $$\frac{d^2Q}{dt^2} + 2m\frac{dQ}{dt} + n^2 Q = \frac{E}{L}$$

[After putting $2m = \frac{R}{L}$ & $n^2 = \frac{1}{LC}$]

or, $$\frac{d^2Q}{dt^2} + 2m\frac{dQ}{dt} + n^2\left(Q - \frac{E}{n^2L}\right) = 0$$

putting $x = Q - \frac{E}{n^2L}$, $\frac{dx}{dt} = \frac{dQ}{dt}$ & $\frac{d^2x}{dt^2} = \frac{d^2Q}{dt^2}$

we get:-

$$\frac{d^2x}{dt^2} + 2m\frac{dx}{dt} + n^2x = 0 \quad \text{--- (1)}$$

Solution of above eqn is

$$x = Ae^{\alpha t}, \text{ i.e } \frac{dx}{dt} = A\alpha e^{\alpha t}$$

and $$\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t}$$

i.e. $A\alpha^2 e^{\alpha t} + 2mA\alpha e^{\alpha t} + n^2 A e^{\alpha t} = 0$

i.e. $\alpha^2 + 2m\alpha + n^2 = 0$

i.e. $\alpha = -m \pm \sqrt{m^2 - n^2}$

putting value of α in general solu^n of (1)

$$x = A e^{[-m + \sqrt{m^2-n^2}]} + B e^{[-m - \sqrt{m^2-n^2}]}$$

where A & B are constants

i.e. $$Q = x + \frac{E}{n^2 L} = \frac{E}{n^2 L} + A e^{(-m+\sqrt{m^2-n^2})t} + B e^{(-m-\sqrt{m^2-n^2})t}$$

$$= EC + A e^{(-m+\sqrt{m^2-n^2})t} + B e^{(-m-\sqrt{m^2-n^2})t}$$

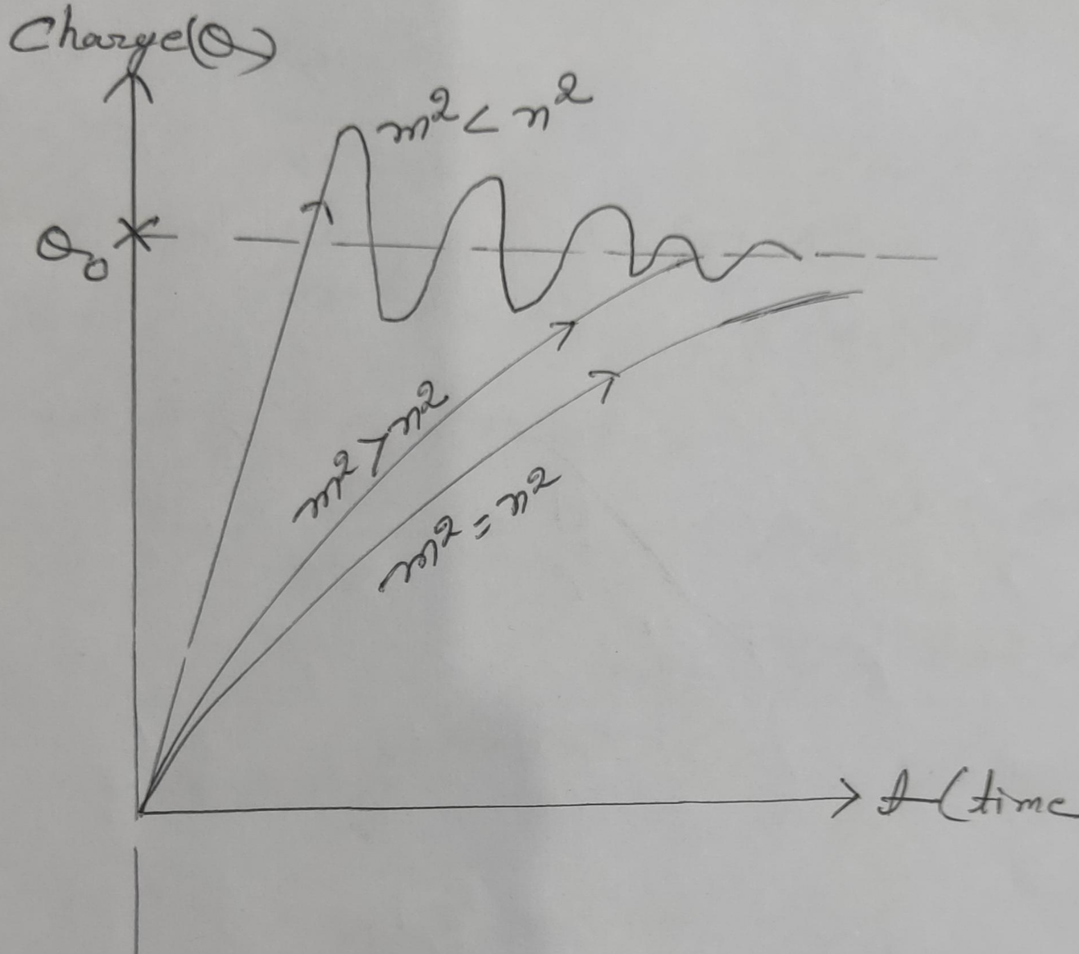
$\left\{ \text{i.e. } n^2 = \frac{1}{LC} \right\}$

$$= Q_0 + A e^{(-m+\sqrt{m^2-n^2})t} + B e^{(-m-\sqrt{m^2-n^2})t} \quad \text{--- (2)}$$

$\{ Q_0 = EC = $ Maxm. charge attained after long time $\}$

Now, differenciating (2) w.r.t. t we have

$$\boxed{\frac{dQ}{dt} = I = \left(-m + \sqrt{m^2-n^2}\right) A e^{(-m+\sqrt{m^2-n^2})}}$$



Growth of current in LCR Circuit